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Abstract

In the past, the analysis of aliasing probability has only dealt with combinational faults, i.e., those that do not convert a combinational circuit into a sequential one. The results obtained so far are valid, for example, for stuck-at faults or a certain class of shorts between lines, but are not valid for delay or stuck-open faults, which are commonly encountered in practice and cause sequential behavior of initially combinational circuits. In general, faults that convert a combinational circuit (or subcircuit) into a sequential one are referred to as sequential faults. In this paper we show that, as the test sequence length is increased, the probability of aliasing for such faults tends to 2^{-k}, where k is the number of binary memory elements in a linear compactor, if the linear compactor has irreducible characteristic polynomial. The results are also valid for circuits under test that are initially sequential. Some recent results on combinational faults are special cases of the results presented in this paper.

Index Terms – aliasing probability, built-in self-test, cellular automaton, linear feedback shift register, signature analysis, test response compaction.

1 Introduction

Built-in self-test (BIST) is one of the most promising solutions to testing problems associated with manufacturing and use of complex VLSI circuits. The underlying idea of

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BIST is to design extra on-chip circuitry that, during a testing session, generates and applies test patterns to the circuit under test (CUT), and analyzes the CUT responses. Usually, for practical reasons, the large amount of test response data produced by a CUT is compacted into a relatively small number of bits.

Several different compaction schemes have been proposed for BIST so far [1, 2, 3, 4, 5, 6, 7, 8]. Each of them uses a finite state machine (FSM), referred to as a *compacting finite state machine* (CFSM). A CFSM is first initialized, and then undergoes a sequence of state transitions, as the sequence of test response vectors produced by the CUT is applied to its inputs. At the end of a testing session, the final state of the CFSM is checked, thereby constituting the basis for a pass/fail analysis. Since the number of storage elements, e.g., flip-flops, of a CFSM is generally small compared with the number of bits in the test response sequence, compaction may imply an information loss, such that a faulty circuit that produces an erroneous test response may yield the same final state as its fault-free counterpart. Such a situation leads to an incorrect pass/fail decision, and is referred to as an *aliasing* error.

The quality of testing is often judged by fault coverage, i.e., the fraction of faults in a particular class that a given test procedure can detect. Obviously, aliasing errors decrease the fault coverage. The fraction of faults that are not detectable due to aliasing is referred to as *aliasing ratio*. For a given test procedure, it can be determined by simulation. Unfortunately, such simulation is generally very expensive, especially for large circuits. The aliasing ratio is, therefore, estimated by a probability, referred to as the *aliasing probability* (see, e.g., [9, 10]).

The determination of the aliasing probability associated with different compaction schemes for BIST has attracted much recent research interest. The major effort has been devoted to studying aliasing that occurs in *signature analysis*, which, due to its simplicity, is the most popular scheme used in practice. Much work has been done on aliasing in single-input signature registers (see, e.g., [11]). Early findings concerning multiple-input signature registers (MISRs) were based on some unrealistic error models used to describe the behavior of a faulty CUT. Some recent results [12, 13, 14] consider error models adequate for VLSI circuits, provided that a fault does not convert a combinational CUT

into a sequential one. In this paper, we generalize these results. We consider aliasing when a faulty CUT constitutes an FSM. Our result applies not only to MISRs but also to a wider class of compactors, referred to as *linear compactors*, which includes, for example, cellular automata.

The error models and results presented in [12, 13, 14] and some other related work are summarized in section 2. The necessity to use a more general error model is discussed in section 3. This section also introduces the error model that we use in this paper. Sec. 4 contains preliminaries, namely, some definitions and results that are used in the subsequent section to prove our main result that is presented in Section 5. Finally, Section 6 concludes the paper.

2 Recent Results

Early research on aliasing in signature analysis was focused on single input signature registers. Numerous papers have been published on this topic over the past several years. A survey of these results can be found, for example, in [11]. Over the last couple of years more and more research efforts have been devoted to studying multiple input signature registers (MISRs) [15, 16, 17, 18, 19, 12, 20, 21, 14]. This trend is natural, since the vast majority of VLSI circuits have multiple outputs, and, therefore, MISRs are of greater practical significance than their single input counterparts.

Many results on aliasing probability for MISRs were obtained under very simplistic assumptions on the behavior of a faulty CUT. In general, to determine aliasing probability, it is assumed that test patterns are generated randomly, and, therefore, the errors that appear at the CUT outputs are characterized by a probability distribution. Some of the error models used in the past ignored correlation between errors at different outputs of the CUT. Others assumed that the error vectors were almost uniformly distributed.

Let us consider a CUT stimulated by test patterns characterized by a probability distribution. For this probability distribution, each fault that does not convert the CUT into a sequential circuit will result in a distribution of error vectors at the outputs of the CUT. This error model was used very recently by two research groups [12, 13]. Both groups independently proved the following: as the test seguence length is increased, the

aliasing probability tends to 2^{-k}, where k is the number of binary memory elements of the linear compactor, provided that the compactor is characterized by an irreducible polynomial and that the "no error" vector has a probability different from zero. Their result was generalized in [14], where the above statement was proved to be valid, if any two error vectors, neither of which need to be the "no error" vector, have probabilities of occurrence different from zero. The error model used in [14] also considers the situation in which exactly one error vector has a probability different from zero. For such error distributions, the test lengths at which aliasing occurs were determined.

The error distribution is different for different faults, and so is the aliasing probability. For some L-CFSMs and some error distributions, the aliasing probability may tend to a value greater than 2^{-k}. This has been observed in [20] and [19]. For some error distributions, it is also possible that for a given test length, the aliasing probability is either zero or one.

To estimate aliasing ratio accurately, one has to determine the aliasing probability for each possible fault and then compute the average. Unfortunately, this task requires exact fault simulation, i.e., a procedure that one tries to avoid. Thus, at present, all we may know about aliasing probability is the fact that, for any fault in the CUT, the asymptotic value of aliasing probability is 2^{-k}. The result presented in [12, 13], and then generalized in [14], identifies linear compactors that posess this property. In [14], for single error distributions, the test lengths for which aliasing is possible are also identified. These results appear to constitute the most fundamental fact on aliasing discovered so far. Nevertheless, the fault model covers only a limited class of possible behaviors of faulty CUTs.

3 Model of a Faulty Circuit Under Test

Physical defects that occur in the structure of a VLSI circuit may change some circuit characteristics. This, in turn, may change the circuit functionality, i.e., the circuit may produce erroneous responses. At the circuit level, physical defects that result in erroneous circuit behavior are described by a fault model. The class of faults that is commonly considered in practice to evaluate testing procedures consists of single stuck-at faults.

For this class of faults, it is assumed that there is only one faulty gate in the circuit, and the behavior of the faulty gate is such that an input or an output of the circuit takes value 0 or 1 regardless of the values of other variables produced by the circuit. Such a simplified fault model applies to a very limited class of possible physical defects. More comprehensive fault models take into account multiple stuck-at faults, shorts between lines, stuck-open faults, and delay faults.

From a testing point of view, there is an important difference between stuck-at faults and delay faults. The former never convert a combinational circuit into a sequential one. Delay faults, in contrast, always cause sequential behavior of a faulty circuit, since the correctness of the circuit response, in general, depends on the sequence of input vectors that have already been applied to the circuit. Shorts between lines may occasionally create a feedback connection within a combinational circuit, changing it into a sequential one. Stuck-open faults often cause sequential behavior of initially combinational circuits. In this paper, faults that convert a combinational circuit into a sequential circuit will be referred to as *sequential faults.*

As described in the previous section, non of the papers known to us analyzes the effects of sequential faults on aliasing. The only exception is a circuit simulation study presented in [22], where a very simple delay fault model is assumed. Since sequential faults are encountered in practice, it is important to extend the research on aliasing by considering sequential behavior of a faulty CUT.

In general, a faulty CUT can be considered as a finite state machine. For a given input vector, the CUT response may or may not be erroneous depending on the CUT's state. Since an input vector may change the CUT's state, the consecutive erroneous responses produced by a faulty CUT are not, in general, statistically independent, even if the consecutive input vectors are.

Any finite state machine, in particular, one describing the behavior of a faulty CUT, has one or more strongly connected groups of states. This means that within such a group any state can be reached from any other state, if an appropriate sequence of input vectors is applied to the CUT inputs. A strongly connected group of states that does not have a transition to any state outside the group will be referred to as a *knot.* A knot, is said

aperiodic if there exists an input sequence length l_0 such that, for any length $l > l_0$, any knot state can be reached from any other knot state by applying an appropriate input sequence of length l. If the states of a knot can be reached during a testing session, the knot is a *reachable knot.*

Assumptions :

In the remainder of this paper, we assume that the faulty CUT can produce at least two different error vectors, once it enters a state of any reachable knot. More precisely, among the states of a knot and input vectors, there exist two different pairs (*state*, *input vector*), such that the error vectors produced by the faulty CUT in both situations are constant and different. This assumption assures that the fault is "permanent", i.e., for any CUT state there exists a sequence of input vectors that results in an erroneous CUT response. It is valid for stuck-open faults, delay faults and shorts between lines in nonredundant circuits. (In the case of delay faults, there is exactly one knot). We also assume that all reachable knots are aperiodic. This assumption is obviously true for stuck-open faults, delay faults and most shorts between lines.

4 Preliminaries

The class of compactors that we consider in this paper contains multiple input signature registers (MISRs) [1, 23] and cellular automata used for BIST [4]. The first of the following two subsections formally defines linear compactors and presents a necessary and sufficient condition for aliasing to occur. The second subsection presents relevant concepts and results of [14] and [24].

4.1 Linear Compactors and Aliasing

We consider the states of a CFSM as elements of a k-dimensional vector space $\Theta_k = \{0,1\}^k$ over the Galois field GF(2) with binary addition "+" and multiplication "$\cdot$".[1] Θ_k thus consists of 2^k vectors. Both the state s of a CFSM and the response r from the CUT belong to Θ_k. In the following, unless otherwise stated, all matrices and vectors

[1] In the rest of the paper, for simplicity, the operator "$\cdot$" will be omitted. Thus, for example, AB will denote $A \cdot B$, and $\mathbf{C}^t$ will denote $\mathbf{C}$ multiplied by itself $t-1$ times.

are over GF(2). Matrix multiplication and vector addition are defined as usual, based on "+" and "·". Symbols 0^k, [0], and [1] denote the all-zero vector, all-zero matrix, and all-one matrix, respectively.

Definition 1 *[14]* A CFSM is called a *linear compacting finite state machine* (L-CFSM) if there exists a $k \times k$ matrix $\mathbf{C}$ such that, for any compactor state s and any test response r, the next state s' of the CFSM is given by $s' = s\mathbf{C} + r$. □

By Definition 1, if the initial state of a L-CFSM is s_0 and a response sequence r_0, r_1, ..., r_{t-1} is applied to it, the resulting states, s_t ($t = 1, 2, \ldots$), can be expressed as follows:

$$s_t = s_0\mathbf{C}^t + \sum_{i=0}^{t-1} r_i\mathbf{C}^i. \tag{1}$$

Every response r_i from the CUT can be expressed as $r_i = f_i + e_i$, where f_i is the error-free response and e_i, is an error vector. For all i, we have $e_i \in \mathcal{E}$, where $\mathcal{E} = \{\epsilon_0, \epsilon_1, \ldots\}$ denotes the set of all possible error vectors. $0^k \in \mathcal{E}$, if the CUT sometimes produces error-free outputs. From (1), it follows that

$$s_t = s_0\mathbf{C}^t + \sum_{i=0}^{t-1}(f_i + e_i)\mathbf{C}^i = s_0\mathbf{C}^t + \sum_{i=0}^{t-1} f_i\mathbf{C}^i + \sum_{i=0}^{t-1} e_i\mathbf{C}^i.$$

Notice that $s_0\mathbf{C}^t + \sum_{i=0}^{t-1} f_i\mathbf{C}^i$ represents the L-CFSM state produced by an error-free CUT response sequence. Thus, we have the following.

Proposition 1 *Let e_0, e_1, ..., e_{t-1} be the error sequence contained in a response sequence of length t. Aliasing error occurs if and only if*

$$\sum_{i=0}^{t-1} e_i\mathbf{C}^i = 0^k \tag{2}$$

and there is an i, $0 \le i \le t-1$, such that $e_i \neq 0^k$. □

4.2 Relevant Notions and Previous Results

To prove our main result, we need to use some results of our earlier papers [14, 24]. First, however, we formally define the notion of characteristic polynomial, as it plays an important role in our considerations.

Definition 2 [25] Given a transition matrix $\mathbf{C}$, the *characteristic polynomial* for $\mathbf{C}$, denoted $\phi(x)$, is given by $|\mathbf{C} - x\mathbf{I}| = 0$, where $\mathbf{I}$ is the identity matrix. □

4.2.1 Markov Process Model

By Proposition 1, to determine if aliasing occurs it is sufficient to consider only the sequences of errors. Thus, given a certain distribution of error vectors fed to the CUT, one can construct the *transition probability matrix*, $\mathbf{P}$. This matrix has n rows and n columns, where $n = 2^k$, i.e., the number of states of the L-CFSM. Entry $p_{i,j}$ of $\mathbf{P}$ is the probability that a transition from state s_i to s_j occurs. This implies that $\mathbf{P}$ is a stochastc matrix, i.e., the sum of all entries in any row equals 1. For a class of L-CFSMs $\mathbf{P}$ is doubly stochastic, i.e., each column of $\mathbf{P}$ also sums to 1. The class of L-CFSMs that have doubly stochastic transition probability matrices, regardless of the particular error vector distribution, is identified by the following lemma.

Lemma 1 *[14] Let $\mathbf{C}$ be the matrix representing a L-CFSM. $\mathbf{C}$ is non-singular (i.e., $|\mathbf{C}| \neq 0$) if and only if, for any state s and for any response vector r, there exists a unique state s' such that r causes a transition from s' to s, i.e., $s = s'\mathbf{C} + r$.*

The results discussed in Section 2 imply that the L-CFSMs which are characterized by irreducible polynomials are the primary candidates to be used in practice. Obviously, for such L-CFSMs, $|\mathbf{C}| \neq 0$ holds.

If a fault does not convert the initially combinational CUT into a sequential circuit, then the distribution of error vectors does not change with time, provided that the test patterns are generated by a random process with a time-invariant distribution. The L-CFSM can then be modeled by a *time-homogeneous Markov process* [26] and the probability of aliasing at time t is determined by the entry of $\mathbf{P}^t$ that represents the probability of t-step transition from state 0^k to 0^k.

If a faulty CUT is a sequential circuit, the error vector distribution varies in time, and a time-homogeneous Markov process cannot be used to model the L-CFSM. At any time instant, however, the distribution of error vectors depends on the CUT state, u, and the probability, $p(s \rightarrow s')$, that the L-CFSM changes its state from s to s', can be

expressed using conditional probabilities. Let $p(s \rightarrow s' \mid u)$ denote the probability of transition $s \rightarrow s'$ provided the CUT is in state u. Then $p(s \rightarrow s') = \sum_{u \in U} p(s \rightarrow s'|u) z_u$, where z_u denotes the probability that the CUT is in state u, and U denotes the set of all CUT's states. Thus, the behavior of the L-CFSM can be described by a t-step transition probability matrix, $\mathbf{M}(t)$,

$$\mathbf{M}(t) = \prod_{i=0}^{t-1} \sum_{u \in U} z_u(i) \mathbf{P}(u), \tag{3}$$

where $z_u(i)$ is the probability that, at time $t = i$, the CUT is in state u, and $\mathbf{P}(u)$ is the transition probability matrix of the L-CFSM when the CUT is in state u. Of course, by Proposition 1, we have $p_{0^k}(0) = 1$. Thus, the time-homogeneous case is a special case where, at any time, t, the probability of aliasing is determined by the entry of $\mathbf{M}(t)$ that represents t-step transition probability from state 0^k to 0^k.

To investigate the behavior of a time-nonhomogeneous process described by equation (3), we use the following result.

Theorem 1 *Suppose that* $\mathbf{C}$ *has an irreducible characteristic polynomial and* $|\mathcal{E}| \geq 2$, *where* $\mathcal{E} = \{\epsilon_0, \epsilon_1, \ldots\}$ *denotes the set of all possible error vectors. Then, for each* $v \in \Theta_k$, *there exists a sequence of error vectors,* $e_0, e_1, \ldots, e_{t-1}$, *such that*

$$\sum_{i=0}^{t-1} (e_i - \epsilon_0) \mathbf{C}^i = v$$

for any fixed $t \geq k$.

Proof: Immediately follows from Theorems 1 and 2 of [14]. □

Although equation (3) describes the behavior of a time-nonhomogeneous process, each matrix $\mathbf{P}(u), u \in U$, may be associated with a time-homogeneous Markov process. Theorem 1, which says that the L-CFSM state transition diagram is strongly connected and aperiodic, implies the following corollary on *ergodicity* [26].

Corollary 1 *[14] Let* $\mathbf{C}$ *be the matrix representing a L-CFSM. The Markov process represented by the transition probability matrix* $\mathbf{P}$ *is ergodic, if the characteristic polynomial of* $\mathbf{C}$ *is irreducible and the error vector set* $\mathcal{E}$ *contains at least two vectors that have probabilities different from zero.* □

It is known that, for any ergodic Markov process represented by a doubly stochastic matrix $\mathbf{P}$, the limiting probabilities of all states are equal [26, page 399], regardless of the initial state probability distribution. In our case the limiting probability of each state is 2^{-k}.

4.2.2 Convergence of a Markov Process

Our goal is to prove that, for the error model introduced in Section 3, the probability of aliasing by L-CFSMs with irreducible characteristic polynomials converges to 2^{-k}, as the test sequence length increases. This can be achieved by proving that the state probability distributions of the corresponding time-nonhomogeneous process described by equation (3) converges to the uniform distribution. To do this we shall use a measure of "distance" between the state probability distribution at time t, and the stationary distribution. We adopt such a measure from [24].

Let $\vec{x}(t) = [x_1(t), \ldots, x_n(t)]$ denote a vector such that $x_i(t)$ is the probability of being in state i at time t. Thus, for $t = 0, 1, \ldots$, we have $\sum_{i=1}^{n} x_i(t) = 1$ and $\vec{x}(t+1) = \vec{x}(t)\mathbf{P}$. The stationary distribution is denoted by $\vec{\pi} = [\pi_1, \ldots, \pi_n]$. We use the measure of a distance between $\vec{x}(t)$ and $\vec{\pi}$ is defined by

$$\delta(\vec{x}(t)) = \max_{1 \le i \le n} |x_i(t) - \pi_i|. \tag{4}$$

To investigate the behavior of $\delta(\vec{x}(t))$ with respect to t, we use the following theorem.

Theorem 2 *[24]* Given the transition matrix $\mathbf{P}$ of any ergodic Markov process, let $\{q_1^{[k]}, \ldots, q_n^{[k]}\}$ be a sorted list of the elements in the k-th column of $\mathbf{P}$, such that for any $i, 1 \le i \le n-1$, $q_i^{[k]} \ge q_{i+1}^{[k]}$. For any probability distribution vector $\vec{x}(t)$,

$$\delta(\vec{x}(t+1)) \le \rho(\mathbf{P})\delta(\vec{x}(t)),$$

where

$$\rho(\mathbf{P}) = \max_{1 \le k \le n} \Big(\sum_{i=1}^{\lfloor n/2 \rfloor} q_i^{[k]} - \sum_{i=\lceil n/2 \rceil + 1}^{n} q_i^{[k]} \Big).$$

□

From the above theorem, it immediately follows that, if $\mathbf{P}$ is doubly stochastic and each column of $\mathbf{P}$ has less than half of its entries equal to zero, then $\rho(\mathbf{P}) < 1$.

5 Main Result

Now, that the necessary notions and results have been presented, we can formulate and prove the main result of this paper.

Theorem 3 *Let* $\mathbf{C}$ *be the matrix representing a L-CFSM. If the characteristic polynomial of* $\mathbf{C}$ *is irreducible and each reachable knot of the CUT is aperiodic and capable of producing at least two different error vectors, then the aliasing probability converges to* 2^{-k}*, as the test sequence length is increased.*

Proof: After sufficiently long sequence of test patterns has been applied to the CUT inputs, the CUT enters a state belonging to a knot, N. Now, for the fault model described in Section 3, the CUT state transition in response to a randomly generated input sequence can be modeled by a Markov process. As every knot is assumed to be aperiodic, the Markov process that describes the CUT is ergodic. This means that the probability, $z_u(t)$, of any state $u \in N$, converges to a stationary probability, π_r, that is different from zero. Thus, for any $\gamma > 0$, after some initial period of time t_0, all states in N have probabilities that differ from thier stationary probabilities by at most γ, i.e.,

$$\forall \gamma > 0 \;\; \exists t_0 \;\; \forall t > t_0 \;\; \forall u \in N \;\; : \quad |z_u(t) - \pi_u| < \gamma.$$

From this observation and the fact that the CUT is capable of producing at least two error vectors, it follows that, for any $t > t_0$, the factor $\sum_{u \in U} z_u(t)\mathbf{P}(u)$ in equation (3) is a matrix corresponding to a transition probability matrix for a distribution that has at least two error vectors with nonzero probability. In fact, these vectors have a nonzero probability for any $t > t_0$, although the probability may vary in time. Thus, by Theorem 1, any state $u \in N$ can be reached from any other state at any time $t \geq k$. This implies that, for any $t > t_0$, matrix $\prod_{j=0}^{k-1} \sum_{u \in U} z_u(t+j)\mathbf{P}(u)$ has all entries different from zero. Since all such matrices are doubly stochastic (each of them is a product of doubly

stochastic matrices), the processes they represent have the same stationary distribution. Thus, by Theorem 2, the distance, $\delta(\vec{z}(t))$, between the L-CFSM state distribution at time t and the stationary distribution, shrinks every k clock cycles. Since, after time $t > t_0$, all CUT states have probabilities in the γ-neighbourhood of their stationary distribution, there exists a minimum probability of k-step transition between any two states.[2] This implies that

$$\exists \rho_0 < 1 \;\; \exists t_0 \;\; \forall t > t_0 \;\; : \quad \rho(\prod_{j=0}^{k-1} \sum_{u \in U} z_u(t+j)\mathbf{P}(u)) < \rho_0.$$

Thus, after some initial time t_0, the distance, $\delta(\vec{z}(t))$, shrinks at least by a factor $\rho_0 < 1$. This means that $\vec{z}(t)$ converges to the stationary distribution and implies that the aliasing probability converges to 2^{-k}. □

6 Conclusion

We have shown that if the characteristic polynomial of a linear compacting finite state machine is irreducible, then, as the test sequence length is increased, the aliasing probability tends to 2^{-k}, where k is the number of binary memory elements in the L-CFSM, provided that any reachable knot of the faulty circuit under test is aperiodic and is capable of producing at least two different error vectors. The result applies to circuits under test that initially were sequential, but, more importantly, it also applies to sequential faults, i.e., faults that convert combinational circuits into sequential circuits (e.g., delay faults or stuck-open faults). Thus, generalizing the previous results derived for combinational faults, we conclude that L-CFSMs with irreducible characteristic polynomials are primary candidates to be used in practice.

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Centre for Systems Science

• Simon Fraser University established the **Centre for Systems Science** as a provincial centre for advanced study. The objective of CSS is twofold: coordinate and combine advanced systems research at SFU, and encourage application of research results within industry in order to further the technological development of British Columbia.

• CSS has assembled three research groups to streamline its efforts. The orientation of these groups is as follows: intelligent systems, microelectronics, and computer and communications systems. Because of shared research interests, this latter group merges with LCCR (**Laboratory for Computer and Communications Research**) for the publication of technical reports.

• CSS's three groups involve over 80 skilled faculty researchers principally from SFU's Schools of Engineering Science and Computing Science, but also draw substantially on Mathematics, Physics, Geography, Philosophy, Linguistics, Psychology, Communication, and Kinesiology.